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Supersymmetric 4D rotating black holes from 5D black rings

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Abstract: We present supersymmetric solutions describing black holes with non-vanishing angular momentum in four dimensional asymptotically flat space. The solutions are obtained by Kaluza-Klein reduction of five-dimensional supersymmetric black rings wrapped on the fiber of a Taub-NUT space. We show that in the four-dimensional description the singularity of the nut can be hidden behind a regular black hole event horizon and thereby obtain an explicit example of a non-static multi-black hole solution in four asymptotically flat dimensions.

Keywords: Black Holes in String Theory.

Contents

JHEP08(2005)042

1. Introduction

It is well-known that when the supersymmetric limit of a four-dimensional Kerr-Newman black hole is taken, one must at the same time set the angular momentum to zero in order to preserve the regular horizon.[1] Thus, no rotating, supersymmetric, single-black hole solutions have been known in asymptotically flat four-dimensional spacetimes.

[1] See, e.g. [1] for a general discussion on the interplay between angular momentum and supersymmetry. Our work indicates that still more surprises with angular momentum remain to be discovered.

On the other hand, supersymmetric rotating black holes *do* exist in five dimensions. Besides the BMPV black hole of [2], with a horizon of spherical topology, recently we constructed a supersymmetric black ring of five-dimensional supergravity [3]. In this paper, we shall show that there is a natural mechanism that reduces these five-dimensional black rings to supersymmetric configurations of four-dimensional black holes with angular momentum. The existence of the configuration that we shall describe has been conjectured by the authors of [4]. A direct construction in four dimensions of supersymmetric multi-black hole solutions with angular momentum has been given earlier in [5].[2]

In a four-dimensional set-up, the basic idea is as follows. Consider a supersymmetric, asymptotically flat black hole solution of $\mathcal{N} = 2$ supergravity which is electrically charged with respect to one of the gauge fields of the theory. If we now place a magnetic monopole, of the same (abelian) gauge field, a finite distance away from the black hole, then angular momentum is generated by the crossed electric and magnetic fields. Properly speaking, the black hole itself is not rotating, since the geometry near the horizon is static,[3] but the configuration is able to carry a non-vanishing angular momentum while preserving four supercharges. It therefore seems appropriate to refer to the system as rotating.

In the simplest form of this configuration, the magnetic monopole gives rise to a naked singularity. So one may worry that such singularities are the price to pay in order to have both rotation and supersymmetry. However, we shall show that our five-dimensional construction can be easily extended to hide away the singularity behind a black hole horizon, and construct a supersymmetric two-black hole configuration with non-zero angular momentum. This is free from naked singularities as well as from other possible pathologies such as closed timelike curves (CTCs).[4]

We obtain the four-dimensional solutions by Kaluza-Klein (KK) reduction of five-dimensional solutions consisting of supersymmetric black rings [3] in a Taub-NUT (TN) space. A rough way to think about this configuration (which is explained more precisely in section 5) is that the S^1 factor of the black ring horizon wraps the S^1 fiber of the TN space, so reduction to four dimensions yields a black hole with an S^2 horizon. The black ring carries two independent angular momenta. One combination of these descends to the angular momentum of the four-dimensional black hole, whereas the other corresponds to momentum along the TN fiber and so becomes the electric charge of the four-dimensional black hole. Although the five-dimensional solution is completely regular (up to a harmless orbifold singularity), the 'nut' of the TN space yields a naked singularity of a KK monopole upon reduction to four dimensions. This can be remedied by placing at the nut a five-

[2]See [6] for an early general study of stationary supersymmetric solutions in four dimensions.

[3]Note that a supersymmetric, asymptotically flat, black hole must have vanishing angular velocity but nevertheless one often refers to such an object as rotating if it is non-static. Examples are the 5D BMPV black hole [2, 7] or the supersymmetric black ring [3].

[4]Such regular, asymptotically flat, non-static multi-black hole solutions apparently do not exist in *minimal* $\mathcal{N} = 2$ supergravity (i.e. Einstein-Maxwell theory). Assuming that any such solution must be supersymmetric, the results of [8] reveal that it would have to belong to the Israel-Wilson-Perjes family of solutions but it has been argued that the only black hole solutions of this family are the static multi-Reissner-Nordstrom solutions [9].

JHEP08(2005)042

dimensional black hole with an S^3 horizon, which is mutually supersymmetric with the black ring. Reduction to four dimensions then yields a solution with two regular S^2 horizons and non-zero angular momentum.

A closely related configuration has been considered in [4] where a two-charge supertube [10], instead of a black ring, wraps the TN fiber. Our configuration can be regarded as an extension of this to three-charge/three-dipole supertubes [11 − 13], which are known to correspond, for equal charges, to the supersymmetric black rings that we consider in this paper. In fact, it should be straightforward to extend our solution, using the methods in [13], to general three-charge supertubes in TN. However, only in the case that the supertube carries all three charges and three dipoles does one obtain a solution with a regular horizon, both in 4D and 5D.

The ability of Taub-NUT space to compactify a five-dimensional black object to a four-dimensional one has also been exploited very recently in [14]. In this case a black hole (essentially a Taub-NUT extension of the BMPV solution) is placed at the nut with the rotation one-form aligned with the TN fiber, so the resulting four-dimensional solution does not carry any angular momentum. One limit of our solutions, obtained by collapsing the ring to the nut, also describes a black hole in Taub-NUT. However, as we shall discuss in section 5.2, it is different than the one in [14].

The rest of the paper is organized as follows. In section 2 we construct the five-dimensional solution describing a supersymmetric black ring in a Taub-NUT background. We study its physical properties in section 3: this includes the near-horizon geometry (section 3.1), the non-trivial asymptotic structure (section 3.2), the conditions for absence of CTCs (section 3.3), and the physical parameters (section 3.4). In section 4 we study briefly two particular limits of the system: the collapse of the ring to a black hole at the nut, and the removal of the nut to infinity, in which the ring becomes a compactified black string. We perform the reduction to four dimensions in section 5 and study the properties of the four-dimensional black hole obtained from the five-dimensional black rings. In section 6 we construct a five-dimensional solution describing, in the Taub-NUT background, a black hole at the nut and a black ring away from the nut. We conclude with a discussion in section 7.

The appendices contain technical details. Appendix A provides the black ring solution in coordinates useful for studying the near-horizon structure, and for connecting the solutions found here with the supersymmetric black rings in asymptotically flat space. In appendix B we prove that the black ring horizon is smooth, and in appendix C we show that provided the parameters obey certain constraints there are no CTCs outside the horizon.

Note added: following the appearance of the first version of this paper, we were informed by F. Denef of his earlier four-dimensional construction of supersymmetric rotating multi-black holes [5], of which we were unaware. Shortly afterwards ref. [15] appeared, which also describes black rings in Taub-NUT and provides the connection between the two constructions. More recently, TN-black rings have also been discussed in [16].

JHEP08(2005)042

2. Black rings in Taub-NUT space

Any supersymmetric solution of five-dimensional minimal supergravity must admit a non-spacelike Killing vector field V [17]. In a region where V is time-like, the metric and gauge potential can be written as [18]

$$ds^2 = -H^{-2}(dt+\omega)^2 + H\, ds^2(\mathcal{B}), \tag{2.1}$$

$$\mathcal{A} = \frac{\sqrt{3}}{2}\left[H^{-1}(dt+\omega) - \beta\right], \tag{2.2}$$

where $V = \partial/\partial t$, the so-called base space $\mathcal{B}$ is an arbitrary hyper-Kähler space, and H is a scalar function while ω, β are one-forms on $\mathcal{B}$. The one-form β is related to ω through the relation $3Hd\beta = d\omega + \star_4 d\omega$, where $\star_4$ is the Hodge star on $\mathcal{B}$.

Supersymmetry implies that H and ω must obey a pair of coupled equations on $\mathcal{B}$. If $\mathcal{B}$ is a Gibbons-Hawking (GH) space [19], i.e, if $\mathcal{B}$ admits a Killing field that preserves the complex structures, then it is straightforward to solve these equations.[5] The general solution involves four harmonic functions on $\mathbb{R}^3$ [18].

Recently, a supersymmetric, asymptotically flat, black ring solution was obtained [3]. The base space of this solution is flat: $\mathcal{B} = \mathbb{R}^4$. It was observed in [21] that this solution can be written in the GH form just discussed. Now, (self-dual, euclidean) Taub-NUT space is also a GH space. This suggests that we should be able to construct a solution describing a supersymmetric black ring in Taub-NUT as a special case of the general GH solution of [18].

The metric of Taub-NUT space is

$$ds^2(\mathrm{TN}) = H_k\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2\right] + H_k^{-1}(dz + Q_k\cos\theta\, d\phi)^2, \tag{2.3}$$

with

$$H_k = 1 + \frac{Q_k}{r} \tag{2.4}$$

and $0 \le \theta \le \pi$. The periodicities of the angular coordinates are $\Delta\phi = 2\pi$ and $\Delta z = 2\pi R_k$. The parameter Q_k is quantized as

$$Q_k = \frac{1}{2}N_k R_k, \tag{2.5}$$

where N_k is an integer. Note that R_k is the asymptotic radius of the z-circle in the base space, but it need not (indeed it will not) be the asymptotic radius in the full solution. Similarly, Q_k will not be exactly the same as the magnetic KK charge. The surfaces of constant r are squashed lens spaces $S^3/\mathbb{Z}_{N_k}$.

The solution (2.1)-(2.2) is specified in terms of three harmonic functions K, L and M on $\mathbb{R}^3$ as follows (the fourth harmonic function is H_k). First, the one-forms β and ω are split as

$$\beta = \beta_0(dz + Q_k\cos\theta d\phi) + \tilde{\beta}, \qquad \omega = \omega_0(dz + Q_k\cos\theta d\phi) + \tilde{\omega}, \tag{2.6}$$

[5]Provided one assumes that the GH Killing field extends to a symmetry of the full 5D solution.

where $\tilde{\beta}$ and $\tilde{\omega}$ are one-forms on $\mathbb{R}^3$. Then H, β_0 and ω_0 are given by [18, 20]

$$\begin{aligned} H &= H_k^{-1} K^2 + L \,, \\ \beta_0 &= H_k^{-1} K \,, \\ \omega_0 &= H_k^{-2} K^3 + \frac{3}{2} H_k^{-1} K L + M \,, \end{aligned} \tag{2.7}$$

whereas $\tilde{\beta}$ and $\tilde{\omega}$ are determined by

$$\begin{aligned} d\tilde{\beta} &= - \star_3 dK \,, \\ d\tilde{\omega} &= \star_3 \left[H_k dM - M dH_k + \frac{3}{2}(K dL - L dK) \right] , \end{aligned} \tag{2.8}$$

where $\star_3$ denotes the Hodge dual on $\mathbb{R}^3$.

Guided by [21], we obtain a supersymmetric black ring in Taub-NUT by choosing the harmonic functions to be

$$K = -\frac{q}{2\Delta} \,, \qquad L = 1 + \frac{Q - 2qv_H Q_k}{4Q_k \Delta} \,, \qquad M = v_H \left(1 - \frac{R}{\Delta} \right) , \tag{2.9}$$

where q, Q and R are constants, Δ is the distance to $\vec{x}_0 = (0, 0, -R)$ in $\mathbb{R}^3$,

$$\Delta = |\vec{x} - \vec{x}_0| = \sqrt{r^2 + R^2 + 2Rr\cos\theta} \tag{2.10}$$

and we have defined

$$v_H = \frac{3q}{4\,(Q_k + R)} \,. \tag{2.11}$$

The parameters R, Q_k, q have dimensions of length and Q has dimensions of length squared. Q and R have the interpretation of the charge of the ring and (a measure of) its distance to the nut, but it should be noted that they do not bear a simple direct relation to the parameters with the same labels in the solution for the ring in asymptotically flat 5D space in [3]. q yields the dipole charge of the ring and is essentially the same parameter as in [3]. The detailed relation between this solution and the one in [3] is given in appendix A. In the next section we shall see that the dimensionless quantity v_H can be interpreted as a velocity of the five-dimensional horizon in the z direction.

We shall assume that

$$Q \geq 2qv_H Q_k \,, \qquad q \geq 0 \,. \tag{2.12}$$

We can now collect these pieces to give the solution we seek. With the above harmonics (2.9) we have

$$H = 1 + \frac{Q - 2qv_H Q_k}{4Q_k \Delta} + \frac{q^2}{4\,H_k\,\Delta^2} \,. \tag{2.13}$$

The one-forms ω and β are found using (2.7) and (2.8). In particular,

$$\omega_0 = v_H \left(1 - \frac{R}{\Delta} \right) - \frac{3q}{4H_k \Delta} \left(1 + \frac{Q - 2qv_H Q_k}{4Q_k \Delta} \right) - \frac{q^3}{8H_k^2 \Delta^3} \tag{2.14}$$

and

$$\tilde{\omega} = \tilde{\omega}_\phi \, d\phi = \frac{2v_H \, Q_k R \, r \, \sin^2\theta}{\Delta \, (r + R + \Delta)} \, d\phi \, . \tag{2.15}$$

β is simply

$$\beta = -\frac{q}{2\Delta} \left[H_k^{-1}(dz + Q_k \cos\theta \, d\phi) - (R + r\cos\theta) d\phi \right] \, . \tag{2.16}$$

Together with (2.3), these provide all the input needed to specify the solution in (2.1), (2.2).

We have made a number of choices of various coefficients in the harmonic functions and of integration constants. The choices are made to ensure that the four-dimensional metric obtained by reduction along ∂_z is asymptotically flat and that the five-dimensional solution possesses no Dirac-Misner strings. These two requirements translate into the conditions that $\tilde{\omega}_\phi \to 0$ as $r \to \infty$ and $\tilde{\omega}_\phi(\theta = 0, \pi) = 0$, respectively. We have furthermore made a simplifying choice by having as well $K \to 0$ as $r \to \infty$, but this can be relaxed. We comment on the possibilities of more general solutions in section 6.3.

3. Physical properties

The solution constructed in the previous section describes a supersymmetric black ring with $S^2 \times S^1$ horizon in a Taub-NUT background. In this section we examine its physical properties as a five-dimensional solution; its reduction to four dimensions will be studied in section 5. First in subsection 3.1 we briefly discuss the near-horizon limit. The details are fairly technical and are given in appendix B, where we find coordinates that extend the metric smoothly through the horizon. In section 3.2 we study the non-trivial asymptotic structure. Next in section 3.3 we comment on how to avoid CTCs (again the details are deferred to appendix C). Finally we compute the physical charges in section 3.4.

3.1 Near-horizon geometry

Near the horizon, the structure of the solution remains basically the same as for the black ring in asymptotically flat 5D of [3]. In the near-horizon limit (defined in appendix B) we find

$$ds^2_{\rm nh} = 2d\tilde{v}d\tilde{r} + \frac{4\hat{L}\tilde{r}}{q} d\tilde{v}d\hat{\psi}' + \hat{L}^2 d\hat{\psi}'^2 + \frac{q^2}{4}\left[d\bar{\theta}^2 + \sin^2\bar{\theta} d\chi^2\right] . \tag{3.1}$$

Here $\tilde{v}$ and $\hat{\psi}'$ are time and angular coordinates regular on the horizon, while $\bar{\theta}$ and χ parametrize a 2-sphere. We have also introduced

$$\hat{L} = \sqrt{3\left[\left(\frac{Q - 2qv_H Q_k}{2q}\right)^2 - \frac{4Q_k^2 R}{R + Q_k}\right]} \, , \tag{3.2}$$

corresponding to the quantity defined in (B.4). The near-horizon limit of the metric describes the product of locally AdS$_3$ of radius q with a two-sphere of radius $q/2$. This is exactly the same as for the asymptotically flat black rings of [3] up to the new expression for $\hat{L}$.

A necessary condition for absence of CTCs near the horizon is obviously $\hat{L}^2 > 0$, i.e.

$$\left(\frac{Q - 2qv_H Q_k}{2q}\right)^2 > \frac{4Q_k^2 R}{R + Q_k}\,. \tag{3.3}$$

This is sufficient to eliminate CTCs to leading order near the horizon. However, in contrast to the situation for asymptotically flat rings [3], this condition alone will *not* be sufficient to avoid causal pathologies in the full solution.

In appendix B we show that both the metric and the inverse metric are analytic at $\tilde{r} = 0$ and we show that the $\tilde{r} = 0$ hypersurface is indeed a Killing event horizon for the black ring. The metric on a spatial cross-section of the horizon is

$$ds^2_{\text{horizon}} = \hat{L}^2 d\hat{\psi}'^2 + \frac{q^2}{4}\left[d\bar{\theta}^2 + \sin^2\bar{\theta}d\chi^2\right], \tag{3.4}$$

so the horizon topology is $S^2 \times S^1$. The area of the horizon is

$$A_{\text{H}} = 2\pi^2 N_k^{-1}\hat{L}q^2 = \frac{\pi^2 q}{N_k}\sqrt{3\left[(Q - 2qv_H Q_k)^2 - 16\frac{q^2 Q_k^2 R}{R + Q_k}\right]}\,. \tag{3.5}$$

In appendix B we show that the black ring horizon is generated by the orbits of the Killing vector

$$\xi = \frac{\partial}{\partial t} \tag{3.6}$$

at constant values of z and ϕ (or $\hat{\psi}$ and $\hat{\phi}$ in (A.4)) on $\tilde{r} = 0$. This implies that the horizon velocity in either of the directions z or ϕ vanishes. However we will see in the next subsection that surfaces of constant z are moving with respect to asymptotic observers at rest, which is in contrast to the case of black rings in asymptotically flat 5d space [3] where the horizon is static.

3.2 Asymptotic structure

The asymptotic structure of the Taub-NUT black rings turns out to be quite non-trivial. When $r \to \infty$, the 5d metric asymptotes to

$$ds^2_{\text{asymp}} = -\Big(dt + v_H(dz + Q_k\cos\theta\, d\phi)\Big)^2 + (dz + Q_k\cos\theta\, d\phi)^2 + d\mathbf{x}_3^2\,. \tag{3.7}$$

The cross-term $dt(dz + Q_k\cos\theta\, d\phi)$ comes from the fact that $\omega_0 \to v_H$ asymptotically. It implies that the asymptotic frame is not at rest. In order to go to a frame that is asymptotically at rest, write first the asymptotic metric as

$$ds^2_{\text{asymp}} = \gamma^{-2}\left[(dz + Q_k\cos\theta\, d\phi) - v_H\gamma^2\, dt\right]^2 - \gamma^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2\,, \tag{3.8}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v_H^2}}\,. \tag{3.9}$$

We shall see shortly that γ can be interpreted as a relativistic dilation due to the horizon moving with velocity v_H with respect to asymptotic observers. We now go to a frame at rest by doing a coordinate transformation. At the same time, we rescale t and z to be, respectively, canonically normalized time and compact Kaluza-Klein direction at infinity:

$$t = \gamma^{-1}\,\bar{t}\,, \qquad z = \gamma\,(\bar{z} + v_H\,\bar{t})\,. \tag{3.10}$$

The asymptotic metric is then

$$ds^2_{\text{asymp}} = \left[d\bar{z} + \frac{Q_k}{\gamma}\cos\theta\,d\phi\right]^2 - d\bar{t}^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\,d\phi^2\,. \tag{3.11}$$

The generator of the horizon (3.6), in terms of the coordinates of asymptotic observers at rest, is

$$\xi = \gamma\left(\frac{\partial}{\partial\bar{t}} - v_H\frac{\partial}{\partial\bar{z}}\right). \tag{3.12}$$

Hence the horizon is moving with velocity v_H along $\bar{z}$ with respect to asymptotic infinity. So γ is actually the usual relativistic dilation factor, which was the motivation for its definition. Note also the explicit relativistic form of the coordinate transformation (3.10).

As can be seen from the metric (3.8), the requirement $v_H < 1$ is necessary in order to avoid closed causal curves near infinity. Observe also from (3.11) that the physical radius R_5 of the five-dimensional Kaluza-Klein circle at infinity, $\bar{z} \sim \bar{z} + 2\pi R_5$, is not that of the base space, R_k, but is contracted by a factor

$$R_5 = R_k\gamma^{-1}\,. \tag{3.13}$$

3.3 Absence of closed timelike curves

In the two previous sections we have found two conditions necessary to ensure that the spacetime is causally well-behaved. The condition

$$v_H = \frac{3q}{4\,(Q_k + R)} < 1 \tag{3.14}$$

is needed in order to avoid closed causal curves near asymptotic infinity. Physically this condition means that the horizon cannot rotate faster than light relative to an observer at rest at infinity.

On the other hand, we have found that in order to avoid CTCs near the horizon, the condition (3.3) must be obeyed. It is shown in appendix C that (3.3) and (3.14) are sufficient conditions to ensure that the spacetime has no CTCs outside the horizon.

3.4 Physical parameters

The physical parameters of the solution can be computed by taking either a five-dimensional viewpoint, in terms of quantities that belong in the minimal five-dimensional supergravity, or in terms of the four-dimensional theory that is obtained via KK reduction. Both kinds of magnitudes are of course directly related (the relation is given in section 5 below), but the five-dimensional theory is simpler and so we take this viewpoint in this section.

JHEP08(2005)042

The spacetime has a translation-invariant direction $\bar{z}$ and hence it can be assigned a 2×2 ADM stress-energy tensor T_{ab}, $a, b = \bar{t}, \bar{z}$. This is computed from the asymptotic metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (in cartesian coordinates) as

$$T_{ab} = \frac{1}{16\pi G_5} \int d\Omega_{(2)} r^2 n^i \left[\eta_{ab} \left(\partial_i h^c_c + \partial_i h^j_j - \partial_j h^j_i \right) - \partial_i h_{ab} \right] \tag{3.15}$$

where n^i is the radial normal vector and a, b, c run over parallel directions $\bar{t}, \bar{z}$ while i, j run over transverse directions. The integration is over the transverse angular directions. Using this we obtain the mass and momentum as the integrated energy and momentum densities,[6]

$$M = \int d\bar{z}\, T_{\bar{t}\bar{t}} = \frac{\pi}{4G_5} \frac{\gamma}{N_k} \left(3Q + 4Q_k^2 (1 - 2v_H^2) + 8 v_H^2 Q_k R \right), \tag{3.16}$$

$$P = \int d\bar{z}\, T_{\bar{t}\bar{z}} = \frac{\pi}{4G_5} \frac{\gamma v_H}{N_k} \left(3Q - 4v_H^2 Q_k^2 + 8 R Q_k \right). \tag{3.17}$$

The electric charge is defined as

$$\mathbf{Q} = \frac{1}{16\pi G_5} \int_S \left(\frac{2}{\sqrt{3}} \star \mathcal{F} - \frac{4}{3} \mathcal{A} \wedge \mathcal{F} \right), \tag{3.18}$$

where S denotes the $S^3/\mathbb{Z}_{N_k}$ at infinity and we have chosen to absorb the awkward factor $2/\sqrt{3}$ to eliminate it from the charge. The presence of the Chern-Simons term is required for the charge to be conserved.[7] This term can be computed in any gauge regular in a neighbourhood of infinity. It is gauge-invariant because the pull-backs to S of any two gauge potentials must differ by an exact form on S (since S has vanishing first cohomology). One must be careful to work in the frame that is at rest at infinity as defined above. We find

$$\mathbf{Q} = \frac{\pi}{4G_5} \frac{Q}{N_k}. \tag{3.19}$$

We define the Kaluza-Klein monopole charge in a manner motivated by its four-dimensional interpretation,

$$\mathbf{Q}_k = \frac{2\pi R_5}{4G_5} \frac{N_k R_5}{2} = \frac{\pi}{4G_5} \frac{4Q_k^2}{N_k \gamma^2}. \tag{3.20}$$

Then we have

$$E_\xi \equiv \gamma \left(M - v_H P \right) = 3\mathbf{Q} + \mathbf{Q}_k, \tag{3.21}$$

which is presumably the BPS relation for this system. Note that the l.h.s. is the energy defined with respect to the supersymmetric (co-moving) Killing field ξ (3.6).

[6]The ubiquitous presence of the factor $\pi/(4G_5)$ can be eliminated by judiciously setting to one the eleven-dimensional Planck constant, $\ell_{11} = (4G_5/\pi)^{1/3} = 1$.

[7]This term vanishes in an asymptotically flat context but it is non-vanishing here.

We can compute the dipole charge using the 5D formula

$$\mathbf{D} = \frac{1}{16\pi G_5}\int_{S^2}\frac{2}{\sqrt{3}}\mathcal{F} = \frac{q}{8G_5}\,, \tag{3.22}$$

where the S^2 links the ring only once (one such surface is obtained at constant $t, y, \hat{\psi}$, in the coordinates introduced in appendix A). Since the topology of the base space is $\mathbb{R}^4$, the black ring does not wrap any non-trivial cycle and it can be stripped of this charge. However, and in contrast to the situation for the asymptotically flat 5D rings, in the Taub-NUT background this dipole charge *is* conserved.[8] As explained in [23], when the source of this charge is unwound a current appears that takes the charge towards the nut. Taub-NUT space possesses a harmonic anti-self-dual two-form [24] that gives rise to a zero-mode of the gauge field. This gets excited in the process of unwinding and as a result the nut acquires the charge.[9]

Finally, in five dimensions the rotation along ϕ gives rise to an angular momentum density $\mathcal{J}_\phi$ which is obtained from

$$-\frac{g_{\bar{t}\phi}}{g_{\bar{t}\bar{t}}} = \frac{2G_5\mathcal{J}_\phi\sin^2\theta}{r} + O(r^{-2}) \tag{3.23}$$

so the angular momentum is

$$\begin{aligned} J_\phi &= \int d\bar{z}\,\mathcal{J}_\phi = \frac{\pi}{G_5}\gamma v_H R R_5 Q_k \\ &= 2v_H\gamma^2 R\mathbf{Q}_k\,. \end{aligned} \tag{3.24}$$

The last expression can be used to eliminate the parameter R (which does not correspond to any invariantly defined quantity) and obtain the relation

$$\frac{1}{2}\mathbf{Q}_k\left(P - 3\gamma v_H\mathbf{Q} + \gamma^3 v_H^3\mathbf{Q}_k\right) = \frac{\pi R_5}{4G_5}J_\phi\,. \tag{3.25}$$

In the limit where the ring and the nut are very far apart, R being much larger than all charges, we have $v_H \to 0$ and $\gamma \to 1$ so this equation becomes,

$$\frac{1}{2}\mathbf{Q}_k P = \frac{\pi R_5}{4G_5}J_\phi\,. \tag{3.26}$$

P and $\mathbf{Q}_k$ are actually quantized. The quantization of the KK magnetic charge has already been given in (3.20),

$$N_k = \frac{4G_5}{\pi}\frac{\mathbf{Q}_k}{R_5^2} \tag{3.27}$$

and the momentum is quantized as

$$N_p = R_5 P\,. \tag{3.28}$$

[8] Hence the term 'dipole' is a bit of a misnomer, which we keep only for its connection with the solutions in [22, 3]. In fact, the field created by this charge in four dimensions behaves at large distances like a monopole field.

[9] It must be noted, though, that for the bound state that the supersymmetric black ring represents, the M5-branes cannot be unwound and separated from the M2-branes without breaking supersymmetry.

JHEP08(2005)042

Then (3.26) is

$$\frac{N_p N_k}{2} = J_\phi\,. \tag{3.29}$$

This is the Dirac formula for the Poynting angular momentum created by the electric and magnetic KK charges. For finite values of R, however, there are additional terms in (3.25) that modify the 'effective electric charge' and which deserve to be better understood.

4. Limits

We study here two limits of the black ring: first the $R \to 0$ limit where the solution describes a spherical black hole at the nut, secondly the limit $R \to \infty$ yielding a compactified black string.

4.1 Black hole at the nut

In the limit $R \to 0$ the black ring collapses to a spherical black hole at the nut. In this case,

$$H = 1 + \frac{Q - \frac{3}{2}q^2}{4Q_k r} + \frac{q^2}{4H_k r^2}\,, \tag{4.1}$$

$$\omega_0 = v_H \left(1 - \frac{Q_k}{H_k r} - \frac{Q - \frac{3}{2}q^2}{4r^2 H_k} - \frac{q^2 Q_k}{6r^3 H_k^2}\right), \tag{4.2}$$

and

$$\tilde{\omega}_\phi = 0\,. \tag{4.3}$$

The latter implies that $J_\phi = 0$ and that ω is aligned with the TN fiber — hence the reduction to four-dimensions results in a static solution. A regular horizon is located at $r = 0$ and there the solution is similar to the BMPV black hole. The latter is actually recovered in the limit $Q_k \to \infty$, $r \to 0$, $z \to \infty$ with new finite coordinates $\rho^2 = 4Q_k r$ and $\psi = z/Q_k$.

For later purposes in this paper it will suffice to discuss in this section the simpler case of a static black hole at the nut. This is obtained by taking $R = 0$ and $q = 0$. Note that $\omega = \beta = 0$ so this is a static electrically charged solution in the Taub-NUT background. The metric is[10]

$$ds^2 = -H^{-2}dt^2 + \frac{H}{H_k}\left(dz + Q_k \cos\theta d\phi\right)^2 + HH_k\left(dr^2 + r^2 d\Omega^2_{(2)}\right) \tag{4.4}$$

with $H = 1 + \bar{Q}/r$ and $\bar{Q} \equiv Q/(4Q_k)$. This is one of the possible uplifts to five-dimensions of the familiar supersymmetric four-charge black hole (with three of the charges set equal to Q), and in particular when $\bar{Q} = Q_k$ it reduces in 4D to the extremal Reissner-Nordström solution. The horizon at $r = 0$ is regular, and the KK circle does not shrink to zero at $r = 0$ but remains finite. Hence $r = 0$ will also remain a regular horizon in 4D, and this will be used later to hide the singularity of the KK monopole in four dimensions.

[10]The same solution is obtained setting $J = 0$ in the solution in [14].

JHEP08(2005)042

Near the horizon the solution becomes locally AdS_2 of radius $\sqrt{Q}/2$ times a lens space $S^3/\mathbb{Z}_{N_k}$ of radius $\sqrt{Q}$. The spatial cross-section of the horizon has topology $S^3/\mathbb{Z}_{N_k}$ and the area is

$$A_H = \frac{2\pi^2 Q^{3/2}}{N_k}\,. \tag{4.5}$$

This is not the same as the $q, R \to 0$ limit of the black ring horizon area given in (3.5). The same phenomenon was discussed in [3, 11], where it was argued to be due to the different topology of the horizons, and is analogous to the discontinuity in the horizon area in the merger of two supersymmetric Reissner-Nordström black holes. Nevertheless the physical charges measured at infinity are correctly obtained by setting $R = q = 0$ in the expressions in section 3.4.

4.2 Compactified black string

In the limit $R \to \infty$ where the nut is moved infinitely far away from the black ring we recover a compactified version of the supersymmetric black string found in [25].

To take this limit it is convenient to choose coordinates in the base space that are centered at the ring instead of at the nut. Denoting these coordinates with a bar, they are

$$\bar{r} = \Delta\,, \qquad \sin\bar{\theta} = \frac{r}{\Delta}\sin\theta\,. \tag{4.6}$$

Then take $R \to \infty$ keeping $\bar{r}$ and $\bar{\theta}$ finite. In this limit $H_k \to 1$ and

$$H \to 1 + \frac{\bar{Q}}{\bar{r}} + \frac{q^2}{4\bar{r}^2}\,, \qquad \omega_0 \to -\frac{3q}{2\bar{r}} - \frac{3q\bar{Q}}{4\bar{r}^2} - \frac{q^3}{8\bar{r}^3} \tag{4.7}$$

where, again, $\bar{Q} \equiv Q/(4Q_k)$. Since $\cos\theta \to -1$, and consequently $(dz + Q_k \cos\theta d\phi) \to (dz - Q_k d\phi)$, it is convenient to gauge-transform the string away, $z \to z + Q_k\phi$. Then one reproduces the solution in [25], compactified on a circle with period $\Delta z = 4\pi Q_k/N_k$.

Nevertheless, the effects of the topology of Taub-NUT on the ring, even at arbitrarily large but finite R, make it significantly different than the compactified black string in which the nut is absent. The ring in Taub-NUT can always be unwound (even if it remains arbitrarily far from the nut [23]) whereas the black string of [25] cannot. This is because the limit above changes the topology of the base space from $\mathbb{R}^4$ to $\mathbb{R}^3 \times S^1$, and the string wraps a non-contractible cycle. Another difference is that the black string has $J_\phi = 0$ whereas for the ring in TN space the angular momentum (3.29) does not approach zero even at arbitrarily large values of R. Again, the reason is topological: Dirac's result for the angular momentum (3.29) is a topological linking number which does not depend on the distance between the electric and magnetic poles.

5. Supersymmetric four-dimensional black holes

In the background of Taub-NUT space, the circle direction along the ring becomes a compact dimension and the solution naturally reduces a la KK to a four-dimensional black hole with angular momentum in the direction ϕ. It would be incorrect, though, to think that this angular momentum results directly from the rotation of the S^2 in the black ring. The ϕ direction of the four-dimensional black hole is *not* the same as the azimuthal angle $\hat{\phi}$ for the S^2 of the black ring (introduced in appendix A), but it is instead $\phi = \hat{\phi} - \hat{\psi}$, where $\hat{\psi}$ is the angular direction along the S^1 of the ring. So, if we want to interpret the rotation of the four-dimensional black hole in terms of the rotation of the five-dimensional black ring, the correct relation is $J_\phi \leftrightarrow J_{\hat{\phi}} - J_{\hat{\psi}}$. On the other hand, since $z = Q_k(\hat{\psi} + \hat{\phi})$, the momentum P actually corresponds to the combination $J_{\hat{\psi}} + J_{\hat{\phi}}$.

In the reduction to four dimensions we get not only a black hole at $r = R$, $\theta = \pi$, which carries, among other charges, a KK electric charge from its motion in the $\bar{z}$ direction. We also obtain a KK magnetic monopole at $r = 0$ from the reduction of the Taub-NUT geometry. Then, from the point of view of four-dimensional physics, it is the presence of both electric and magnetic KK charges that gives rise to rotation in four dimensions.

The KK monopole is a naked singularity from the four-dimensional point of view. This, however, is not an essential difficulty since we know that in five dimensions the curvature singularity disappears (leaving only a well-understood $\mathbb{Z}_{N_k}$ orbifold singularity). Moreover, in four dimensions this singularity can be hidden by placing a black hole at the origin of the Taub-NUT space, as we will show in the next section. The basic physics of the solution, though, is already exhibited in the solution with a naked nut, which we analyze now.

5.1 Reduction of the 5D action

The nutty black rings are solutions of the equations of motion obtained from the action of $\mathcal{N} = 1$ $D = 5$ minimal supergravity,

$$S_{5\mathrm{d}} = \frac{1}{16\pi G_5}\left[\int d^5x\sqrt{-g_{(5)}}\Big(R_{(5)} - \mathcal{F}^2\Big) - \frac{8}{3\sqrt{3}}\int \mathcal{F}\wedge\mathcal{F}\wedge\mathcal{A}\right], \tag{5.1}$$

where $\mathcal{F} = d\mathcal{A}$. Writing

$$ds^2_{5\mathrm{d}} = e^{-2\Phi/\sqrt{3}}(dz + C_\mu dx^\mu)^2 + e^{\Phi/\sqrt{3}}ds^2_{4\mathrm{d}}\,, \tag{5.2}$$

so the four-dimensional metric is in the Einstein frame, and $\mathcal{A} = A_\mu dx^\mu + \rho\, dz$, we find for the four-dimensional action

$$\begin{aligned} S_{4\mathrm{d}} = \frac{1}{16\pi G_4}\Bigg[&\int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\Phi)^2 - 2e^{2\Phi/\sqrt{3}}(\partial\rho)^2 - \frac{1}{4}e^{-\sqrt{3}\Phi}G^2 - e^{-\Phi/\sqrt{3}}\tilde{F}^2\right) - \\ &- \frac{8}{\sqrt{3}}\int \rho\, F\wedge F\Bigg]. \end{aligned} \tag{5.3}$$

where

$$G_4 = \frac{G_5}{(2\pi R_5)}\,, \qquad G = dC\,, \qquad F = dA\,, \qquad \text{and} \qquad \tilde{F} = F + C\wedge d\rho\,. \tag{5.4}$$

The Chern-Simons term in $\tilde{F}$ comes from the inverse five-dimensional metric.

A consistent truncation of this theory is obtained by setting

$$\rho = 0 = \Phi\,, \qquad \star G = +\frac{2}{\sqrt{3}}F\,, \tag{5.5}$$

which reduces (5.3) to the Einstein-Maxwell theory.

5.2 The 4D solution

The five-dimensional metric can be written in the form

$$ds_{5\mathrm{d}}^2 = \left(\frac{U}{HH_k}\right)^2 \Big[dz - S\,dt + T\,d\phi\Big]^2 + d\tilde{s}_{4\mathrm{d}}^2 \tag{5.6}$$

where

$$d\tilde{s}_{4\mathrm{d}}^2 = -\frac{HH_k}{U^2}\Big[dt + \tilde{\omega}_\phi\,d\phi\Big]^2 + HH_k\Big[dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta d\phi^2\Big]\,, \tag{5.7}$$

with

$$U = \sqrt{H^3 H_k - \omega_0^2 H_k^2}\,, \tag{5.8}$$
$$S = \omega_0 H_k^2 U^{-2}\,, \tag{5.9}$$
$$T = Q_k\cos\theta - \omega_0\tilde{\omega}_\phi H_k^2 U^{-2}\,. \tag{5.10}$$

We introduce now the asymptotic five-dimensional coordinate $\bar{z}$ and the canonical time coordinate $\bar{t}$ defined in (3.10). Then

$$ds_{5\mathrm{d}}^2 = \gamma^2\left(\frac{U}{HH_k}\right)^2\Big[d\bar{z} + \left(v_H - \gamma^{-2}S\right)\,d\bar{t} + \gamma^{-1}T\,d\phi\Big]^2 + d\tilde{s}_{4\mathrm{d}}^2\,, \tag{5.11}$$

hence

$$e^{-\Phi/\sqrt{3}} = \gamma\frac{U}{HH_k}\,, \tag{5.12}$$

the Einstein metric is

$$ds_{4\mathrm{d}}^2 = -\frac{1}{\gamma U}\Big[d\bar{t} + \gamma\tilde{\omega}_\phi\,d\phi\Big]^2 + \gamma U\Big[dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta d\phi^2\Big]\,, \tag{5.13}$$

and the gauge field C has components

$$C_{\bar{t}} = v_H - \gamma^{-2}S\,,\qquad C_\phi = \gamma^{-1}T\,. \tag{5.14}$$

The five-dimensional gauge potential $\mathcal{A}$ gives rise to the four-dimensional pseudoscalar (axion) $\rho = \mathcal{A}_{\bar{z}}$ given by

$$\rho = \frac{\sqrt{3}}{2}\gamma\left(H^{-1}\omega_0 - \beta_0\right) \tag{5.15}$$

and the gauge potential A, taking into account the shift (3.10),

$$A_{\bar{t}} = \frac{\sqrt{3}}{2\gamma}\Big(H^{-1} + v_H\gamma^2\left(H^{-1}\omega_0 - \beta_0\right)\Big)\,,$$
$$A_\phi = \frac{\sqrt{3}}{2}\Big(H^{-1}(\omega_0\,Q_k\cos\theta + \tilde{\omega}_\phi) - (\beta_0\,Q_k\cos\theta + \tilde{\beta}_\phi)\Big)\,. \tag{5.16}$$

The mass and angular momentum (measured in the Einstein metric) are the same as M in (3.16) and J_ϕ in (3.24). The KK electric and magnetic charges correspond to P and $\mathbf{Q}_k$,

$$\mathbf{Q}_e = \frac{1}{16\pi G_4}\int \star\, e^{-\sqrt{3}\Phi} G = P\,, \qquad \mathbf{Q}_m = \frac{1}{16\pi G_4}\int G = \mathbf{Q}_k\,. \tag{5.17}$$

The charge $\mathbf{Q}$ in (3.18) is the same as

$$\mathbf{Q} = \frac{1}{16\pi G_4}\int_{S^2}\left(\star\,\frac{2}{\sqrt{3}}e^{-\Phi/\sqrt{3}}\tilde{F} + \frac{4}{3}(\rho F - d\rho\wedge A)\right). \tag{5.18}$$

The magnetic dual of this charge is

$$\mathbf{P} = \frac{1}{16\pi G_4}\int_{S^2}\frac{2}{\sqrt{3}}\left(\tilde{F} - \rho G\right) = \frac{q}{8G_4} = 2\pi R_5 \mathbf{D}\,, \tag{5.19}$$

which makes explicit that the dipole charge is, in this system, a conserved charge as explained in section 3.4.

Limit $R \to 0$. We have argued in section 4.1 that in the limit $R \to 0$ we obtain a black hole at the nut which, for $q > 0$, is non-static in five-dimensions but static in four.

A related, but different, black hole was considered in [14]. In five dimensions it also describes a BMPV-like black hole at the nut in a TN background, and it is also obtained using the methods of [18]. Using our notation, it corresponds to taking, for K, L, M in (2.7), the functions

$$K = 0\,, \qquad L = 1 + \frac{Q}{4Q_k r}\,, \qquad M = v_H H_k \tag{5.20}$$

with $H_k = 1 + Q_k/r$. When reduced to four dimensions, this is a static black hole with non-zero charges $\mathbf{Q}_e$, $\mathbf{Q}_m$ and $\mathbf{Q}$, but, since $\tilde{\beta} = 0$, it has zero magnetic charge $\mathbf{P}$. This charge distinguishes it from our solution.

6. Rotating BPS two-black hole solutions

The goal of this section is to construct a solution with two black holes: in five dimensions we place a black hole at the nut and a black ring away from the nut. In four dimensions this gives a supersymmetric configuration with two black holes and a net angular momentum. This solution is regular with no singularities outside the horizons.

More generally, one can construct multi-concentric supersymmetric black ring configurations in Taub-NUT, just as it has been done for asymptotically flat black rings [21, 13]. When all the rings wrap the TN fiber, the reduction to 4D describes a sequence of supersymmetric black holes along the axis of rotation to one side of the nut.

6.1 Solution

We shall construct the simplest example of a black hole at the nut and a black ring away from the nut. Take

$$K = -\frac{q}{2\Delta}\,, \qquad L = 1 + \frac{Q_{\rm r} - 2qv_H Q_k}{4Q_k\Delta} + \frac{Q_{\rm h}}{4Q_k r}\,, \qquad M = v_H\left(1 - \frac{R + \frac{Q_{\rm h}}{4Q_k}}{1 - \frac{Q_{\rm h}}{4Q_k^2}}\frac{1}{\Delta}\right), \tag{6.1}$$

where $Q_{\rm r}$ and $Q_{\rm h}$ are constants and

$$v_H = \frac{3q}{4\,(Q_k + R)}\left(1 - \frac{Q_{\rm h}}{4Q_k^2}\right). \tag{6.2}$$

The constants in M are fixed by requiring that $\tilde{\omega}_\phi$ vanishes as $r \to \infty$ (necessary for asymptotic flatness in four dimensions) and that $\tilde{\omega}_\phi(\theta = 0, \pi) = 0$ (to avoid Dirac-Misner strings in five dimensions).

The parameters $Q_{\rm r}$ and $Q_{\rm h}$ play the roles of the charge parameters for the ring and the hole, respectively. When $Q_{\rm h} = 0$ we recover the solution for a black ring in TN of section 2, while $Q_{\rm r} = 0 = q$ yields the static black hole at the nut of section 4.1.

With the above choice, we have

$$H = 1 + \frac{Q_{\rm h}}{4Q_k\, r} + \frac{Q_{\rm r} - 2qv_H Q_k}{4Q_k\Delta} + \frac{q^2}{4H_k\Delta^2}\,, \tag{6.3}$$

$$\omega_0 = v_H\left(1 - \frac{R + \frac{Q_{\rm h}}{4Q_k}}{1 - \frac{Q_{\rm h}}{4Q_k^2}}\frac{1}{\Delta}\right) - \frac{3q}{4H_k\Delta}\left(1 + \frac{Q_{\rm h}}{4Q_k\, r} + \frac{Q - 2qv_H Q_k}{4Q_k\Delta}\right) - \frac{q^3}{8H_k^2\Delta^3}\,, \tag{6.4}$$

and

$$\tilde{\omega}_\phi = \frac{2v_H\, Q_k\, R\, r\, \sin^2\theta}{\Delta\,(r + R + \Delta)}\,. \tag{6.5}$$

Finally, β takes the same form as in (2.16). The solution is then obtained by plugging these functions into (2.1), (2.2) and (2.6).

6.2 Properties

Asymptotics and physical charges

It was shown in section 3.2 that a coordinate transformation is needed to bring the asymptotic metric to a frame at rest. For the two-black hole solution here this is done just as in section 3.2, but now with v_H as given in (6.2). The necessary condition for avoiding CTCs near asymptotic infinity is $v_H < 1$.

The dipole charge of the black ring is $\mathbf{D} = q/(8G_5)$. For the conserved charge (3.18) we find

$$\mathbf{Q} = \frac{\pi}{4G_5 N_k}\left[Q_{\rm h} + Q_{\rm r}\right]. \tag{6.6}$$

Note, however, that it is not rigorous to associate $Q_{\rm h}$ and $Q_{\rm r}$ with the charges of the black hole and the black ring separately, since only the total charge $\mathbf{Q}$ is measured at infinity. Presumably, one could use the method of [26] to assign separate charges to the hole and the ring.

Near-horizon geometry. The analysis of the near-horizon geometries goes through for both the black hole at the nut and the black ring just as for the single-black hole solutions studied in the previous sections.

The black hole at the nut has horizon $S^3/\mathbb{Z}_{N_k}$ with radius $\sqrt{Q_{\rm h}}$. The black ring has an $S^1 \times S^2$ horizon. The radius of the S^2 is $q/2$ and the S^1 has proper length $\hat{L}_r$, where

$$\hat{L}_r = \sqrt{3\left(\frac{(Q_{\rm r} - 2qv_H Q_k)^2}{4q^2} - \frac{Q_k(4RQ_k + Q_{\rm h})}{R + Q_k}\right)}\,. \tag{6.7}$$

qThe horizon areas are

$$A_H^{\rm h} = \frac{2\pi^2 Q_{\rm h}^{3/2}}{N_k}\,, \qquad A_H^{\rm r} = \frac{2\pi^2 q^2 \hat{L}_r}{N_k}\,. \tag{6.8}$$

Absence of CTCs. To avoid CTCs near asymptotic infinity, we must require $v_H < 1$, i.e.

$$3q\left(4Q_k^2 - Q_{\rm h}\right) < 16Q_k^2\left(Q_k + R\right). \tag{6.9}$$

Near the black hole horizons the absence of CTCs requires

$$Q_{\rm h} > 0\,, \qquad Q_{\rm r} - 2qv_H Q_k \geq 2q\sqrt{\frac{Q_k(4RQ_k + Q_{\rm h})}{R + Q_k}}\,. \tag{6.10}$$

The three conditions in equations (6.9) and (6.10) are necessary for absence of CTCs outside the horizons. We have not proven that these conditions are sufficient. However, our proof that there are no CTCs for $Q_{\rm h} = 0$ actually implies that the eigenvalues of the relevant 2×2 matrix are strictly positive provided the appropriate bounds are strictly enforced, i.e. provided these bounds are not saturated. By continuity, it follows that those eigenvalues remain positive for small enough $Q_{\rm h}$, and hence there are no CTCs for a small enough black hole at the nut.

Reduction to four dimensions. The reduction of the black hole + black ring solution yields two black holes in four dimensions, one at $r = 0$, the other at $r = R$. The explicit form of the metric and other fields can be obtained using the same formulas as in section 5.2. The horizons are smooth and the nut singularity is hidden behind the horizon of one of the two black holes. The area of the horizons are easily derived from the five-dimensional areas given above.

The most exciting feature of this solution is that it has a net angular momentum, $G_4 J_\phi = \frac{1}{2} Q_k R v_h \gamma$. This is an explicit example of a supersymmetric four-dimensional multi-black hole configuration with angular momentum and without naked singularities nor any other pathologies.

6.3 Generalizations

A general starting point for finding a solution with a black hole at the nut and a black ring away from the nut is given by

$$K = k_1 + \frac{k_2}{\Delta} + \frac{k_3}{r}\,, \qquad L = l_1 + \frac{l_2}{\Delta} + \frac{l_3}{r}\,, \qquad M = m_1 + \frac{m_2}{\Delta} + \frac{m_3}{r}\,. \tag{6.11}$$

Above we have studied for simplicity the case $k_1 = m_3 = 0$, but one could consider keeping these parameters. In particular, it is interesting that when $k_1 \neq 0$ one can obtain rings with vanishing horizon velocity v_H. To see this, consider the generalization of the solutions of section 2 with $k_3 = l_3 = m_3 = 0$ (i.e. with a naked nut) but $k_1 \neq 0$. A straightforward extension of our analysis in section 2 shows that if we require $\tilde{\omega}_\phi$ not to have Dirac-Misner strings and to vanish at $r \to \infty$, then the conditions

$$m_2 = -m_1 R \tag{6.12}$$

and

$$k_1 l_2 - k_2 l_1 = \frac{2m_1}{3}(R + Q_k) \tag{6.13}$$

must be imposed. The asymptotic behavior of ω_0 implies that the velocity of the horizon is

$$v_H = m_1 + \frac{3}{2}k_1 l_1 + k_1^3 \,. \tag{6.14}$$

Asymptotic flatness also demands that $\gamma U \to 1$ as $r \to \infty$ (with U defined in (5.8) and γ in (3.9)) and this translates into

$$l_1 + k_1^2 = 1 \,. \tag{6.15}$$

For generic v_H, these conditions leave three of the six parameters l_i, k_i, m_i arbitrary. The explicit form of $\tilde{\omega}_\phi$ is obtained by changing $v_H \to m_1$ in (2.15).

Near the horizon we find the same structure as in (3.1) but now

$$\hat{L}^2 = \frac{Q_k^2}{k_2^2}\left(3l_2^2 - 8k_2 m_2\right), \tag{6.16}$$

and the radius of the S^2 is $|k_2|$ instead of $q/2$. Absence of causal pathologies requires $0 \leq v_H < 1$ and $\hat{L}^2 > 0$.

Our choice of $k_1 = 0$ in section (2) was made for simplicity and served our purpose of building a black ring in TN space. But if $k_1 \neq 0$ we can have a non-static configuration (i.e. $m_1, m_2 \neq 0$) with $v_H = 0$. Imposition of (6.12), (6.13), (6.14), (6.15) leads to consistent solutions, but we have not dwelt more on them.

Note furthermore that, besides the constraints imposed by absence of CTCs, the parameters must be such that the mass be positive. When $v_H = 0$ one finds

$$M = \frac{1}{4G_4}\left(3l_2 + 6k_1 k_2 + Q_k(1 - 3k_1^2)\right) \tag{6.17}$$

and it is straightforward to check the BPS relation $M = 3\mathbf{Q} + \mathbf{Q}_k$. Presumably, (3.21) extends to the generic case with $0 < v_H < 1$.

7. Discussion

We have constructed explicit solutions showing that five-dimensional black rings are naturally related to supersymmetric configurations with black holes and angular momentum in four dimensions. The solutions exhibit a number of unusual features which we have only begun to analyze. One of these is the fact that the horizon, despite being supersymmetric, is generically moving at non-zero subluminal speed $0 < v_H < 1$. This introduces a number of peculiar relations among parameters, such as equation (3.21), which we conjecture should be understood as a BPS relation, and the relation (3.25). The study of the larger family of solutions described in section 6.3, which also allow for $v_H = 0$, will probably shed light on the full significance of these issues.

There is an interesting, simpler, related system, that exhibits similar physics but without supersymmetry: a configuration consisting of an electric and a magnetic KK black hole separated apart — or, in string theory terms, a system of a D0-charged black hole and a D6-charged black hole away from it. Again, a non-vanishing electromagnetic angular momentum will be present. The KK electric black hole would uplift to five dimensions as a neutral black ring (like the original black ring in [27]) which wraps the fiber of TN space, while the magnetic black hole would uplift to a black hole sitting at the nut, or to a naked nut in the extremal limit. It would be quite interesting to find explicitly this solution of five-dimensional vacuum gravity.[11] For this configuration, however, all of the supersymmetry is broken, even when the extremal limit is taken for the black holes.

An obvious generalization of the solutions in this paper is to unequal charges, in general to a $U(1)^N$ theory, using a straightforward application of the analysis in [13]. These are the Taub-NUT analogues of the three-charge supersymmetric black rings found in [3, 12, 13]. Although the latter exhibit continuous non-uniqueness (classically) of black rings, this is not the case for the solutions in Taub-NUT. As we have explained, the dipole charges responsible for the non-uniqueness in the 5D asymptotically flat solutions become conserved charges in the Taub-NUT background. Upon reduction to four dimensions they are conserved magnetic charges which, together with the other charges, determine uniquely the black hole solution.

It would be interesting to find a microscopic description, along the lines of [29], of the black ring in Taub-NUT in the limit where the ring is very close to the nut. Then it effectively looks like a ring in a $\mathbb{Z}_{N_k}$-orbifold of 5D space and its physical parameters are simply rescaled by appropriate factors of N_k (see [14]). However, the microscopic picture in [29] includes a zero-point contribution to the oscillator number that apparently does not scale in the correct manner.

Further insight into these solutions may also be obtained by studying three-charge/three-dipole supertube probes [11] in the Taub-NUT background. Finally, other interesting extensions of this work include the microscopic analysis of the system as a D1-D5-P configuration following [30, 4], and the application to these solutions of the attractor mechanism for black rings [31].

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[11]Such neutral solutions in five-dimensions (and also generalizations of them to include dipole charges like in [22]) should also be interesting to provide seeds for the construction of near-extremal excitations of supertubes in Taub-NUT, extending the work of [28].

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A. Taub-NUT black ring in (x, y)-coordinates

The black ring in asymptotically flat 5D space was originally given in [3] using a convenient set of coordinates that foliate space with ring-shaped surfaces. These can be introduced as well for Taub-NUT rings, and in fact they are useful in order to analyze the solution near the horizon. They also provide a simple way to recover the asymptotically flat solution of [3]. These (x, y)-coordinates are introduced through

$$r = -R\frac{x+y}{x-y}\,, \qquad \cos\theta = -1 + 2\frac{1-x^2}{y^2-x^2} = 1 - 2\frac{y^2-1}{y^2-x^2}\,, \tag{A.1}$$

so that

$$\Delta = \frac{2R}{x-y}\,. \tag{A.2}$$

A form of the solution that is more closely related to the one in [3] can be obtained as follows. It is convenient to introduce

$$F_k = \frac{R}{Q_k}\frac{y+x}{y-x}\,H_k = 1 + \frac{R}{Q_k}\frac{y+x}{y-x}\,. \tag{A.3}$$

The function F_k encodes the effect of the Taub-NUT background on the ring. Near the horizon, which lies at $y \to -\infty$, it will simply induce a constant rescaling of magnitudes, but near asymptotic infinity, at $y \to x \to -1$, it diverges and hence has a crucial effect, namely the compactification to finite radius of the z circles.

The angular variables that parametrize the S^1 and the azimuth of the S^2 of the ring in [3] are not the same as z and ϕ. We define new angular variables $\hat{\psi}$, $\hat{\phi}$, as

$$\hat{\phi} = \frac{1}{2}\left(\frac{z}{Q_k} + \phi\right), \qquad \hat{\psi} = \frac{1}{2}\left(\frac{z}{Q_k} - \phi\right), \tag{A.4}$$

which are identified as

$$(\hat{\psi}, \hat{\phi}) \sim (\hat{\psi}, \hat{\phi} + 2\pi) \sim \left(\hat{\psi} + \frac{2\pi}{N_k}, \hat{\phi} + \frac{2\pi}{N_k}\right). \tag{A.5}$$

We also define

$$\hat{R} \equiv 2\sqrt{Q_k R}\,, \qquad f_k \equiv \lim_{y\to-\infty} F_k = 1 + \frac{\hat{R}^2}{4Q_k^2}\,, \qquad \hat{Q} \equiv Q - \frac{q^2}{2f_k^2}\left(1 + \frac{3\hat{R}^2}{4Q_k^2}\right). \tag{A.6}$$

$\hat{R}$ and $\hat{Q}$ correspond to the radius scale and the charge, respectively, that were denoted by R and Q in [3]. We find

$$H = 1 + \frac{\hat{Q} - q^2/f_k^2}{2\hat{R}^2}(x-y) - \frac{q^2}{4\hat{R}^2}\frac{x^2-y^2}{F_k}\,. \tag{A.7}$$

The base space takes on a symmetrical form

$$ds^2(\text{TN}) = \frac{\hat{R}^2}{(x-y)^2}\left[F_k\left(\frac{dy^2}{y^2-1}+\frac{dx^2}{1-x^2}\right)+\frac{1}{F_k}\left((y^2-1)d\hat{\psi}^2+(1-x^2)d\hat{\phi}^2\right)+\right.$$
$$\left.+\left(F_k-F_k^{-1}\right)\frac{(y^2-1)(1-x^2)}{y^2-x^2}\left(d\hat{\psi}-d\hat{\phi}\right)^2\right]. \tag{A.8}$$

In these coordinates the one-form ω has components

$$\omega_{\hat{\psi}} = Q_k\omega_0(1-\cos\theta)-\tilde{\omega}_\phi\,, \qquad \omega_{\hat{\phi}} = Q_k\omega_0(1+\cos\theta)+\tilde{\omega}_\phi\,. \tag{A.9}$$

These are

$$\omega_{\hat{\psi}} = \frac{3q}{2f_k}(1+y)-\frac{3q}{8\hat{R}^2F_k}(y^2-1)\left[\hat{Q}-\left(\frac{q}{f_k}\right)^2-\frac{q^2}{3F_k}(x+y)+\frac{\hat{R}^4}{f_kQ_k^2}\frac{x}{(x-y)^2}\right] \tag{A.10}$$

and

$$\omega_{\hat{\phi}} = -\frac{3q}{8\hat{R}^2F_k}(1-x^2)\left[\hat{Q}-\left(\frac{q}{f_k}\right)^2-\frac{q^2}{3F_k}(x+y)+\frac{\hat{R}^4}{f_kQ_k^2}\frac{x}{(x-y)^2}\right]. \tag{A.11}$$

Finally, the one-form β is

$$\beta = \frac{q}{2}\left[(1+x)\left(1+(F_k^{-1}-1)\frac{1-x}{y-x}\right)d\hat{\phi}+(1+y)\left(1+(F_k^{-1}-1)\frac{y-1}{y-x}\right)d\hat{\psi}\right] \tag{A.12}$$

Here an appropriate gauge choice has been made so that $\beta_{\hat{\phi}}$ vanishes at $x=-1$ and $\beta_{\hat{\psi}}$ at $y=-1$.

Consider now the limit $Q_k\to\infty$ with fixed $\hat{R}$, $\hat{Q}$, q, x and y, i.e. $F_k, f_k\to 1$. Then the metric (A.8) becomes the $\mathbb{R}^4$ base space in the form used for black rings in asymptotically flat spacetime (with $N_k = 1$ and identifying $\hat{R},\hat{\psi},\hat{\phi}$ with the corresponding unhatted variables in [3]). Indeed, it is straightforward to check that the complete solution (2.1) and (2.2) reproduces in this limit the supersymmetric black ring in [3].

B. Near-horizon geometry

Using the solution in (x,y)-coordinates as introduced in the previous appendix, we study the near-horizon geometry $y\to-\infty$.

Near the horizon, define $\bar{r} = -\hat{R}/y$ and $\cos\bar{\theta} = x$. Then the divergences near the horizon are removed by defining new coordinates

$$dt = dv - B(\bar{r})\,d\bar{r}\,, \qquad d\hat{\phi} = d\hat{\phi}' - C(\bar{r})\,d\bar{r}\,, \qquad d\hat{\psi} = d\hat{\psi}' - C(\bar{r})\,d\bar{r}\,, \tag{B.1}$$

with

$$B(\bar{r}) = B_0 + \frac{B_1}{\bar{r}} + \frac{B_2}{\bar{r}^2}\,, \qquad C(\bar{r}) = C_0 + \frac{C_1}{\bar{r}} \tag{B.2}$$

and B_i and C_i are constants to be determined. For the gauge field, we find

$$\begin{aligned}\mathcal{A} = &\frac{2\sqrt{3}f_k}{q^2}\bar{r}^2\Big(1+O(\bar{r})\Big)dv + \frac{\sqrt{3}q}{4}\left(\cos\bar{\theta} + a_0 + O(\bar{r})\right)d\hat{\psi}' - \\ &-\frac{\sqrt{3}q}{4}\Big(1+\cos\bar{\theta}+O(\bar{r})\Big)d\hat{\phi}' + \left(\frac{b_1}{r} + b_0 + O(\bar{r})\right)d\bar{r}\,,\end{aligned} \tag{B.3}$$

where a_0, b_0, and b_1 are constants. The $1/\bar{r}$-divergence in $\mathcal{A}_{\bar{r}}$ can be removed by a gauge transformation.

The $1/\bar{r}$ and $1/\bar{r}^2$ divergences in $g_{\hat{\psi}'\bar{r}}$ and $g_{\bar{r}\bar{r}}$ are cancelled by choosing $B_2 = q^2\hat{L}/(4\hat{R})$ and $C_1 = -q/(2\hat{L})$, where

$$\hat{L} = \sqrt{3\left(\frac{(\hat{Q}-(q/f_k)^2)^2}{4q^2} - \hat{R}^2 f_k^{-1}\right)}\,. \tag{B.4}$$

This leaves a $1/\bar{r}$-divergence in $g_{\bar{r}\bar{r}}$ which can been avoided by setting

$$B_1 = \frac{\hat{Q}+2(q/f_k)^2}{4\hat{L}} + f_k\frac{\hat{L}\,(\hat{Q}-(q/f_k)^2)}{3\hat{R}^2} + \frac{3q^2\hat{R}^2}{16Q_k\hat{L}f_k^2}\,. \tag{B.5}$$

With the above choices, $g_{\bar{r}\bar{r}}$ is a linear function of x at $\bar{r}=0$. The constants C_0 and B_0 can be chosen such that this linear function vanishes and we have $g_{\bar{r}\bar{r}} \to O(\bar{r})$ when $\bar{r}\to 0$.

We can now write the near-horizon geometry

$$\begin{aligned}ds^2_{\text{nh5d}} = &-\frac{16f_k^2\bar{r}^4}{q^4}dv^2 + 2\frac{\hat{R}}{\hat{L}}dvd\bar{r} + \frac{4\bar{r}^3\sin^2\bar{\theta}}{q\hat{R}}d\hat{\phi}'dv + \frac{4\hat{R}\bar{r}}{q}d\hat{\psi}'dv + \frac{3q\bar{r}\sin^2\bar{\theta}}{\hat{L}f_k}d\bar{r}d\hat{\phi}' + \\ &+2\left(\frac{q\hat{L}}{2\hat{R}}\cos\bar{\theta} + \frac{3q\hat{R}}{2\hat{L}f_k} + \frac{\hat{Q}-(q/f_k)^2}{6\hat{R}\hat{L}f_k}\Big[3\hat{R}^2(1+f_k) - 4f_k(2-f_k)\hat{L}^2\Big]\right)d\hat{\psi}'d\bar{r} + \\ &+\hat{L}^2d\hat{\psi}'^2 + \frac{q^2}{4}\Big[d\bar{\theta}^2 + \sin^2\bar{\theta}(d\hat{\psi}'-d\hat{\phi}')^2\Big] + \cdots\end{aligned} \tag{B.6}$$

where "..." denotes subleading terms $O(\bar{r})$, including the leading $O(\bar{r})$-terms in $g_{\bar{r}\bar{r}}$. We note that in the $Q_k\to\infty$ limit we recover the near-horizon geometry of the asymptotically flat black rings of [3].

Going back to $x=\cos\bar{\theta}$, the determinant of this metric is $-q^4\hat{R}^2/16$ at $\bar{r}\to 0$, so both the metric and the inverse metric are analytic at $\bar{r}=0$. The Killing vector ∂_v is null at $\bar{r}=0$ and is normal to the surface defined by $\bar{r}=0$. So the $\bar{r}=0$ null hypersurface is indeed the Killing event horizon for the black ring.

In the near-horizon limit we take $\epsilon\to 0$ while rescaling $\bar{r}$ and v as $\bar{r}=\tilde{r}\hat{L}\epsilon/\hat{R}$ and $v=\tilde{v}/\epsilon$. The metric in this limit is

$$ds^2_{\text{nh}} = 2d\tilde{v}d\tilde{r} + \frac{4\hat{L}\tilde{r}}{q}d\tilde{v}d\hat{\psi}' + \hat{L}^2d\hat{\psi}'^2 + \frac{q^2}{4}\left[d\bar{\theta}^2 + \sin^2\bar{\theta}d\chi^2\right]. \tag{B.7}$$

We have denoted $\chi = \hat{\psi}'-\hat{\phi}' = \hat{\psi}-\hat{\phi}$. The near-horizon limit of the metric describes the product of locally AdS_3 of radius q with a two-sphere of radius $q/2$. This is the same as was obtained for the ring in asymptotically flat 5D space in [3], up to the substitution of $\hat{L}$ for L. In the limit $Q_k\to\infty$ the results of [3] are recovered.

C. Absence of closed timelike curves

A sufficient condition for the metric (2.1) with Taub-NUT base space to be free of closed timelike curves is that the two-dimensional z-ϕ metric

$$ds^2 = H^{-2}[A\,(dz + Q_k \cos\theta\, d\phi)^2 + B\, d\phi^2 - 2C\,(dz + Q_k \cos\theta\, d\phi)\, d\phi] \qquad \text{(C.1)}$$

be positive-definite, where

$$\begin{aligned} A &= H^3\, H_k^{-1} - \omega_0^2\,, \\ B &= H^3\, H_k\, r^2 \sin^2\theta - \tilde{\omega}_\phi^2\,, \\ C &= \omega_0\, \tilde{\omega}_\phi\,. \end{aligned} \qquad \text{(C.2)}$$

H and ω_0 are specified in equations (2.7) in terms of K, L and M, which, together with $\tilde{\omega}_\phi$, are given in equations (2.9), (2.15).

The metric (C.1) is positive-definite if and only if the matrix

$$\begin{pmatrix} A & -C \\ -C & B \end{pmatrix} \qquad \text{(C.3)}$$

is positive-definite. This in turn is equivalent to the two conditions

$$A > 0\,, \qquad AB - C^2 = H^3 H_k^{-1}(A\, H_k^2\, r^2\, \sin^2\theta - \tilde{\omega}_\phi^2) > 0\,, \qquad \text{(C.4)}$$

so it is enough to show that

$$D \equiv A\, H_k^2\, r^2\, \sin^2\theta - \tilde{\omega}_\phi^2 > 0\,. \qquad \text{(C.5)}$$

In the limit $r \to \infty$ we find

$$D = r^2 \sin^2\theta\, \left(1 - v_H^2\right) + \mathcal{O}(r)\,, \qquad \text{(C.6)}$$

so as found in section 3.2 a necessary condition to avoid CTCs asymptotically is that $v_H < 1$.

The near-horizon condition for avoiding CTCs is $\hat{L}^2 > 0$, i.e. from (B.4)

$$\hat{Q} - \tilde{q}^2 > 4\, \tilde{q}\, \sqrt{f_k Q_k R}\,. \qquad \text{(C.7)}$$

where we have defined $\tilde{q} = q/f_k$ for convenience in this appendix, and we are using the quantities $\hat{Q}$ and f_k which were introduced in (A.6).

We now show that these two necessary conditions are also sufficient to ensure absence of CTCs. We begin by noting that D is a monotonically increasing function of $\hat{Q}$. Indeed, since $\hat{Q}$ only enters through L, and we have $dL/d\hat{Q} > 0$, we just need to see that $dA/dL > 0$. A direct calculation yields

$$A = H_k^{-2}\left(\frac{3K^2L^2}{4} - 2K^3M\right) + H_k^{-1}\left(L^3 - 3KLM\right) - M^2\,, \qquad \text{(C.8)}$$

and hence

$$\frac{dA}{dL} = \frac{3}{2} H_k^{-2} K^2 L + 3 H_k^{-1} \left(L^2 - KM\right), \tag{C.9}$$

where

$$L^2 - KM = 1 + \frac{3\, f_k\, \tilde{q}^2 + 4(\hat{Q} - \tilde{q}^2)}{8\, Q_k\, \Delta} + \frac{(\hat{Q} - \tilde{q}^2)^2 - 6\, \tilde{q}^2 f_k\, R\, Q_k}{16\, Q_k^2\, \Delta^2}. \tag{C.10}$$

Both summands in dA/dL are positive provided the bound (C.7) holds. Thus we just need to show that $D > 0$ when $\hat{Q} - \tilde{q}^2 = 4\tilde{q}\sqrt{f_k Q_k R}$.

In this case one finds

$$16\, Q_k^4\, \Delta^3\, D = D_0 + (1 - x^2) \sum_{i=1}^{8} D_i\,, \tag{C.11}$$

where

$$\begin{aligned}
D_0 &= 12\, \tilde{q}^2\, Q_k^4\, (1 + x)\, \Delta \left[3\, \Delta\, (r + R - \Delta) + (1 - x)\, r\, (r + R)\right], \\
D_1 &= 3\, r^2 \tilde{q}^4 (R + Q_k)^3\,, \\
D_2 &= 48\, r\, Q_k^3\, \tilde{q}\, \Delta^2\, (Q_k + r)\, \sqrt{f_k Q_k R}\,, \\
D_3 &= 2\, \tilde{q}^3\, R\, Q_k\, r^2\, (9\Delta + 2R)\, \sqrt{f_k Q_k R}\,, \\
D_4 &= 2\, \tilde{q}^3\, Q_k^3\, r\, (9\Delta + 3r - R)\, \sqrt{f_k Q_k R}\,, \\
D_5 &= 2\, \tilde{q}^3\, Q_k^2\, r\, \left[9(r + R)\Delta + 5R\, r - R^2\right] \sqrt{f_k Q_k R}\,, \\
D_6 &= Q_k^2\, r (r + Q_k)\, \Delta^3\, (16\, Q_k^2 - 9\tilde{q}^2)\,, \\
D_7 &= 12\, \tilde{q}^2 Q_k^2\, R\, r^2\, \Delta\, (2R + 3\Delta)\,, \\
D_8 &= 3\, \tilde{q}^2\, Q_k^3\, r\, \Delta \left[4R^2 + 12 R\, r + 6\Delta\, (3R + r) - 3\Delta^2\right].
\end{aligned} \tag{C.12}$$

Each $D_i \geq 0$. In particular $D_6 \geq 0$ by virtue of the bound $v_H < 1$. D_4 and D_5 are seen to be positive by considering the cases $r < R$ and $r > R$ separately.

We conclude that $D \geq 0$ whenever the two conditions (3.14) and (3.3) hold. Thus, within this range of parameters, there are no CTCs outside the horizon.

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JHEP08(2005)042

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